Lab 6

One Dimensional Collision

A. Purpose

To test Law of Conservation of Momentum and Energy by carts on the track with Arduino and CoolTerm.

B. Introduction

For a single object, momentum $\mathbf{p}$ is defined to be the product of its mass and velocity:

$$\mathbf{p} = m\mathbf{v} \tag{1}$$

In one dimension, it is essential to specify the sign of the momentum and the direction of travel to be positive or negative.

A collision is an impact between two or more objects in a system during a time where no external forces act and only internal forces operate between two or more objects. The net or total momentum of the system remains unchanged. In fact, in three dimensions, it is conserved separately in each direction since momentum is a vector:

$$\sum_i \overrightarrow{P_i} = \overrightarrow{P_x} + \overrightarrow{P_y} + \overrightarrow{P_z} = \overrightarrow{P} \tag{2}$$

Energy is also a conserved quantity in an isolated system. However, kinetic energy, K, is not conserved in general since it is just one form of energy. Instead, the internal energy should be taken into consideration so that the total energy remains constant:

$$\sum_i K_i + U_{\text{int}} = \sum_i \frac{1}{2} m_i v_i^2 + U_{\text{int}} = \text{constant } E \tag{3}$$

where U_{int} is the total internal energy of the system.

Specifically, collisions can be either elastic, where both momentum and kinetic energy are conserved, or inelastic, where only momentum is conserved. Moreover, for the collision between two bodies, there is a limiting case of inelastic collision where the two bodies coalesce after impact. The degree to which a collision is elastic or inelastic is quantified by the coefficient of restitution C_R, a value that normally ranges between zero and one, where one for an elastic collision and zero for a completely inelastic collision. The coefficient of restitution C_R is

$$\text{Coefficient of restitution} \equiv \left| \frac{\text{Relative velocity after collision}}{\text{Relative velocity before collision}} \right| \quad \text{or} \quad C_R \equiv \left| \frac{\mathbf{v_{rel}} \cdot n}{\mathbf{u_{rel}} \cdot n} \right| \tag{4}$$

Though C_R does not explicitly depend on the masses of the objects, it is important to note that the final velocities are mass-dependent. For two- and three-dimensional collisions of rigid bodies, the velocities used are the components perpendicular to the tangent line/plane at the point of contact, i.e. along the line of impact. Besides, with C_R, one can find the equations for a twobody collision. That is,

$$\begin{cases} \mathbf{v_a} = \dfrac{m_a\mathbf{u_a} + m_b\mathbf{u_b} + m_b C_R\left(\mathbf{u_b} - \mathbf{u_a}\right)}{m_a + m_b} = \mathbf{u_c} - \dfrac{\mu}{m_a} C_R \mathbf{u_{rel}} \\ \mathbf{v_b} = \dfrac{m_a\mathbf{u_a} + m_b\mathbf{u_b} + m_a C_R\left(\mathbf{u_a} - \mathbf{u_b}\right)}{m_a + m_b} = \mathbf{u_c} + \dfrac{\mu}{m_b} C_R \mathbf{u_{rel}} \end{cases} \tag{5}$$

where $\mathbf{v_a}, \mathbf{v_b}$ are the final velocity of the first and the second object after impact, $\mathbf{u_a}, \mathbf{u_b}$ are the initial velocity of the two objects before impact, m_a, m_b are their mass, $\mathbf{u_c}$ is the center-of-mass velocity, $\mathbf{u_{rel}}$ is the relative velocity between the bodies before collision, and μ is the reduced mass with the definition $1/\mu = 1/m_a + 1/m_b$.

This experiment tests the principles and studies the kinetic energy of the cart before and after the collision.

C. Apparatus and software

	
Slide Cart and track	Arduino Mega Case + CoolTerm

D. Procedures

1. Pre-lab assignments (hand in before the experiment)
   (1) Download driver for Arduino Mega Case[1] and CoolTerm[2]
   (2) (Important!) See the following link for obtaining data by Arduino Mega Case
   http://www.phys.nthu.edu.tw/~gplab/file/Common%20equipment/Arduino%20Mega.ppt
   (3) Make a flowchart of this experiment and answer the questions
      (i) Prove that eq (5) can be rewritten as

$$\begin{cases} \mathbf{v_a} = \left(1 + C_R\right)\mathbf{u_c} - C_R\mathbf{u_a} \\ \mathbf{v_b} = \left(1 + C_R\right)\mathbf{u_c} - C_R\mathbf{u_b} \end{cases}$$

from the perspective of the center of mass of the two bodies. Do not simply substitute one thing for another; instead, start from conservation laws. Moreover, give a physical interpretation of the symmetric relations. **(Hint: Change the velocities to the frame of the center of mass first, and think about this: during**

[1] To install the driver for Arduino Mega Case: https://www.arduino.cc/en/software
[2] To install CoolTerm: http://freeware.the-meiers.org

the collision, the center of mass is like a fixed wall. From your experience, what would happen if a ball hit a fixed wall?)

(ii) We know that momentum conservation during the collision is guaranteed by Newton's 2nd and 3rd laws. Show that the momentum transfer Δp experienced by each particle in a completely inelastic collision is half that in an elastic collision, and give a physical picture to explain the relation. (Hint: Evaluate the final kinetic energies K' in terms of the original momenta p_A, p_B and momentum transfer Δp, and after finding the relation between K' and K_0, compare the solutions to Δp of the two conditions.)

(iii) In the experiment, we use the Arduino Mega Case to obtain time versus position[3] of the carts during the collision. Consider an experiment that Cart 1 of mass $m_1 = 785.49$ g is made to slide right to left and collide Cart 2 of mass $m_2 = 377.32$ g initially at rest. The experimental result is as shown in Fig. 1.

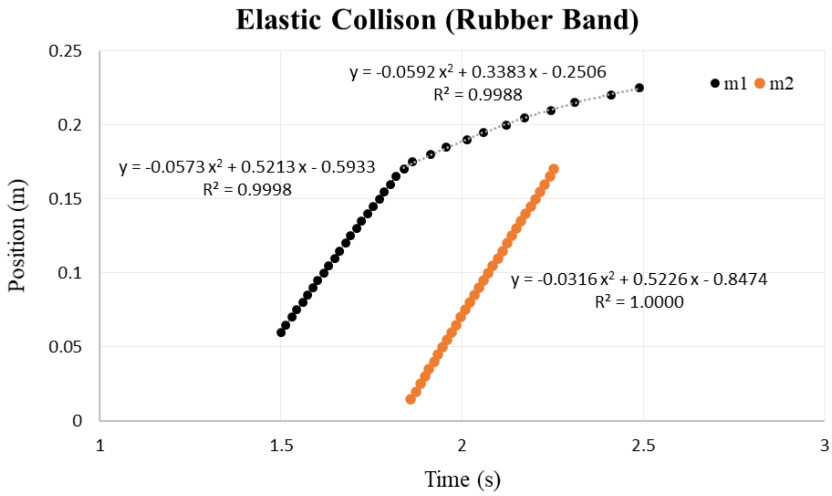

Fig. 1:Elastic Collision (Rubber Band). Equations of the quadratic fitting curves are shown besides the data points, where the black points represent the positions of Cart 1, and the orange points stand for those of Cart 2.

(a) Why do we use quadratic fit to the data? Can we instead use linear fit to do so? What is the difference? Explain.

(b) Find the velocities, the momentums, and the kinetic energies of the two Carts before and after the collision. Also, find the value of C_R.

(Hint: Use the fitting curves to determine the time when two bodies collide with each other.)

(c) Calculate the relative change of the total momentum $D_P(\%) = \left(P_f - P_i\right)/P_i$, and the relative change of the total kinetic energy $D_K(\%) = \left(K_f - K_i\right)/K_i$. Compare the result with the theory.

[3] For convenience, in Fig. 1, the vertical axis is chosen to be positions of the carts.

2 In-lab activities

(1) Make sure the track is horizontal by the level.

(2) Measurement of the widths of Black and White stripes

During the experiment, the sensor on the cart detects the Black and White stripes on the track, and every time the cart goes across the boundaries of a stripe $(x_1 \text{ or } x_2)$, the sensor detects a signal and Arduino would restore the time $(t_1 \text{ or } t_2)$ as shown in Fig.2. This tells the importance of the accurate value of the stripe widths. Measure the widths of the Black and White stripes by your ruler, 20 times for the black stripes, and 20 times for the white ones, and state the result in the standard form $(x = x_{\text{best}} \pm \delta x)$. Note that the widths of black stripes or white stripes are set to be $0.5\ \text{cm}$.

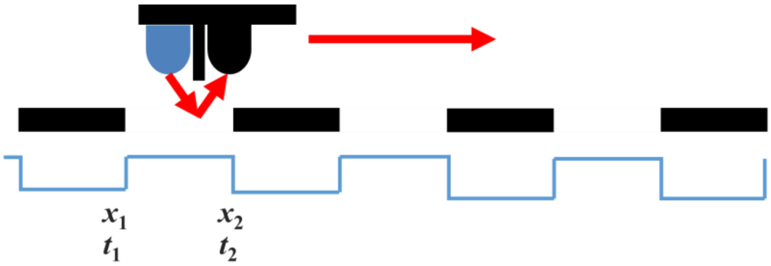

Fig. 2: Data Sampling.

(3) Elastic Collisions

The experiment will be done with cart 2 initially at rest as described in the pre-lab assignment. You will do 6 trials with Trials 1,2,3 denoting the measurement via rubber band bumper accessory and Trials 4,5,6 by the magnet accessory. The choices of m_1, m_2 of each trial are:

Trial 1,4: No extra weight on both cart 1 and cart 2 ($m_1 \cong m_2$)

Trial 2,5: 200 g weight on cart 1 ($m_1 > m_2$)

Trial 3,6: 200 g weight on cart 2 ($m_1 < m_2$)

To use Arduino Mega Case with CoolTerm,

(i) Connect the Arduino Mega Case to laptop and run CoolTerm, and change the function of the Arduino case to **FUNCTION 2**.

(ii) Use the transmission lines to connect the sensor on the carts with the Arduino Mega Case via Signal 1 and 2 by their colors:

*** white→**OUT**, red →**VCC**, and black →**GND*****

(iii) Push **RUN** to start tracking, release the cart and stop it after the collision for at least 2 seconds, and data will be collected in CoolTerm.

(iv) Copy the data from CoolTerm to the data sheet in Excel to store it.

(v) Calculate velocities, momentums, the coefficient of restitution C_R, and kinetic energies of the carts before and after the collision, and compare the result with the theory.

(vi) Push **RESET** to remove data so as to move on to the next trial.

***** Note that there are two Dupont Lines attached to the two carts. Make sure they do not cause unexpected systematic effects during the measurement.**

***** To obtain enough data points, let the distance between two carts be at least 30 cm, and make sure the initial speed of cart 1 is high enough.**

(4) Completely Inelastic Collision

Attach clay-needle accessory to the carts so that they stick together after the collision. Three trials are needed for this part, and the choices of mass for each trial are the same as those for the elastic collision. The process is the same as mentioned in the last section. Compare the results with the theory.

3. Post-lab report
   (1) Recopy and organize your data from the in-lab tables in a neat and more readable form.
   (2) Analyze the data you obtained in the lab and answer the given questions
   (3) Compare the results for two different elastic collisions (Rubber band and Magnet).

E. Questions

1. For the completely inelastic collision, plot the kinetic energy fractional change $D_K(\%) = \left(K_f - K_i\right)/K_i$ versus $x = m_1/(m_1 + m_2)$. What are the slope and intercept of a straight line fit to these data? Does a straight line fit these data reasonably and as expected?
2. The widths of black stripes or white stripes are set to be $0.5\ \text{cm}$. Are the widths same as the best estimate you obtained? If not, the systematic effect occurs during the measurement. How will it affect the result? **Extra credits will be given, if you can answer quantitatively.**

F. Reference

Millet, L. Edward. "The one-dimensional elastic-collision equation: ν f= 2ν c− ν i." The Physics Teacher 36.3 (1998): 186-186.

da Silva, MF Ferreira. "Meaning and usefulness of the coefficient of restitution." European journal of physics 28.6 (2007): 1219.

Wong, Kin Son, and Hang Wong. "Understanding the Law of Conservation of Momentum in One-Dimensional Collisions Between Two Objects Using the Velocity Space Approach." The Physics Teacher 60.2 (2022): 94-96.

Hu, Hui. "More on one-dimensional collisions." The Physics Teacher 40.2 (2002): 72-72.